Heat transfer regimes in mantle dynamics using the CitcomCU software

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This paper was prepared for presentation during the 15th International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 31 July to 3 August, 2017.

Abstract

There are mainly two ways to approach mathematically mantle dynamics: analytically and numerically. The absence of known solutions for the fluid mechanics theory in most of the scenarios makes the analytical approach much challenging to be used in mantle studies. With this in mind, this work presents a study based on the two different main mechanisms responsible for heat transference in a fluid: conduction and convection. Considering an inelastic, incompressible and viscous fluid with infinity Prandtl number in a closed hexahedron, thirty-six simulations were carried to test how well the numerical approach can approximate to the analytical result. With the Rayleigh number and a hexahedron with known geometry, it is possible to predict if the main heat transfer mechanism is through conduction or convection. This work present a good match between the theory and the numerical solution, proving the analytical approach can be well reproduced in a discretized way with the help of a numerical code.

Introduction

The necessity of understanding the Earth's mantle through mathematical approach is very important. Most of what is said about geological activity is related to the mantle somehow. There are two main ways to approach geodynamics mathematically: the analytical and the numerical. Both approaches rely on a set of governing equations. The difference between these approaches is how the equations are solved and, therefore, the difficulty in finding these solutions will define if the analytical or the numerical approach is more suited for one or another scenario.

Generally, the mantle is a very complex subject, and it cannot be easily understood analytically because of the absence of known solutions for the governing equations. That makes mantle dynamics to be usually studied through numerical approach. In this present work, the mantle is represented by an inelastic, incompressible and viscous fluid within a hexahedron. Using the fluid mechanics theory for mantle dynamics, this study will predict the main heat transfer mechanism as a function of the Rayleigh Number (Ra), the geometry of the hexahedral cell and the wavelength of the added temperature perturbation. The prediction is compared to the results of a numerical simulation. The numerical code used for this study was the 3D finite element parallel code CitcomCU from the Computational Infrastructure for Geodynamics (CIG) team modified from the original Citcom code from Moresi and Gurnis (1996).

There are two main ways for heat to be transferred within the mantle: conduction and convection. The heat transfer mechanism is a fluid occurs mainly through convection when the Rayleigh Number Ra is greater than the critic value shown in equation 1. Below the Ra_{CR} curve, the fluid should transfer heat mainly through conduction.

$$Ra_{CR}(b,\lambda) = \frac{\left(\pi^2 + \left(\frac{2\pi b}{\lambda}\right)^2\right)^3}{\left(\frac{2\pi b}{\lambda}\right)^2} \quad (1)$$

where b is the total height of the cell and λ is the wavelength of the temperature perturbance that was added (equation 2) adapted from Turcotte and Schubert (2002).

$$T^*(x,z) = T_0^* \cos\left(\frac{\pi z}{b}\right) \cos\left(\frac{2\pi x}{\lambda}\right) \quad (2)$$

where T^* is the non-dimensional temperature, T_0^* is the amplitude of the temperature perturbance and x and z are the plane directions horizontal and vertical, respectively. Note that the y direction is not considered, and $\partial T/\partial y = 0$ for this study.

Figure 1 shows the graph of Ra_{CR} as a function of $2\pi b/\lambda$, where the blue line represents 50% contribution of each heat transfer mechanism. It is important to say that this Ra_{CR} divides the percental contribution of the heat transfer mechanisms and it does not exclude one or other het transfer mechanism, so a pair ($2\pi b/\lambda$,Ra) above the Ra_{CR} curve represents a scenario where the convection is the main heat transfer mechanism, but it does not imply it is the only mechanism.

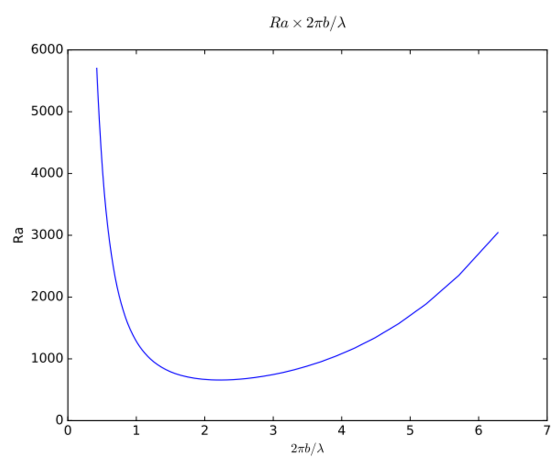

***Figure 1** – Relation between the Rayleigh number and the wavelength λ of the thermal perturbation in the fluid.*

Method

For the simulations to be relevant for a heat transfer study, several $(Ra, 2\pi/\lambda)$ pairs were chosen considering Figure 1 and equation 1. Succinctly, these pairs were chosen to be distributed close to the Ra_{CR} curve for six values of $2\pi b/\lambda$. The simulations were made for geometries where the x-length d would match the wavelength λ, keeping the z-length h fixed with the value b. This implies $d = \lambda = [4\pi b, 2\sqrt{2}b, 2\pi b/3, \pi b/2, 2\pi b/5, \pi b/3]$. Figure 2 shows the pair $\mathrm{d} = \lambda = 2\sqrt{2}b$ and the adopted coordinate system for the equation 2 to be used.

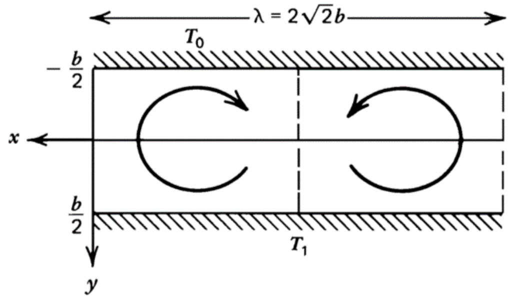

***Figure 2** – Bimodal convection cell with $\Delta x = \lambda = 2\sqrt{2}b$, adapted from Turcotte and Schubert (2002).*

Thirty-six simulations were made. Considering only the initial behavior of the maximum value of the non-dimensional velocity modulus in each simulation, it is possible to verify if the temperature perturbance increases or decreases with time. An increase in this non-dimensional velocity modulus ($|v|$) suggests a predominant convection transfer mechanism, while a decrease suggests a predominant conduction transfer mechanism.

This mechanism can be observed in the initial time-steps of the simulation, so the presents results of this study contain only the first 400 steps of the simulation. The real time in these simulations is not important for the study and all the results are in a non-dimensional time that the CitcomCU code uses (Moresi and Gurnis, 1996).

The initial thermal configuration for each simulation were produced using a Python 2.7 script. For each one of the simulations, the chosen grid had $(49\times49\times49)$ nodes, where (x, y, z) represents the number of nodes in each direction, x, y and z. For each grid, a non-dimensional vertical gradient was generated, where $T_0 = 0.0$ at the top and $T_1 = 1.0$ at the bottom. The gradients $\partial T/\partial x = \partial T/\partial y = 0.0$ and $\partial T/\partial z = (T_1 - T_2)/b$. After that, T^* was summed as a function of x and z for each different λ (as in equation 2). The amplitude of the thermal perturbation was $T_0^* = 2.86\times10^{-4} = 1/3500$, where 3500 is the approximate contrast, in Kelvin, of the temperature between the surface and the core-mantle boundary. An exaggerated initial configuration is similar to Figure 3, for $T_0^* = 0.1$.

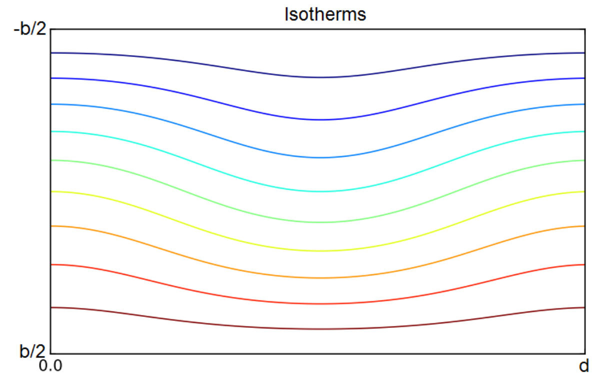

***Figure 3 –** Exaggerated initial thermal configuration for the simulations with $T_0^* = 0.1$.*

For the simulation to run, the viscosity of the fluid was set to be constant and the fluid was also non-rotating with an infinite Prandtl Number in a hexahedral cell. As the temperature gradient in the y direction is null, the non-dimensional velocity modulus in that direction is also null. This implies that the calculations could be made in a single $x\times z$ plane. The CitcomCU output of non-dimensional velocities allows us to calculate the non-dimensional velocity modulus $|v|$ as in equation 3.

$$|v| = \sqrt{u_x^2 + u_z^2} \qquad (3)$$

where u_x and u_z are the non-dimensional velocity values of each node in each x and z direction, respectively. Simulations were divided in groups for each one of the $2\pi b/\lambda$ values, therefore, six, groups. The maximum of the non-dimensional velocity modulus $|v'|$ is the value that will lead to a conclusion about the heat transfer mechanism.

Results

The following profiles are the results for the simulations made with the CicomCU code for each length d and wavelength λ. All the profiles show the variation of the maximum of the non-dimensional velocity modulus through the CitcomCU time t'. The transformation from t' to seconds can be checked in the manual of the CitcomS code (Tan et al., 2014), another Citcom variation, and is not important for the type of analysis that will be carried out in this work.

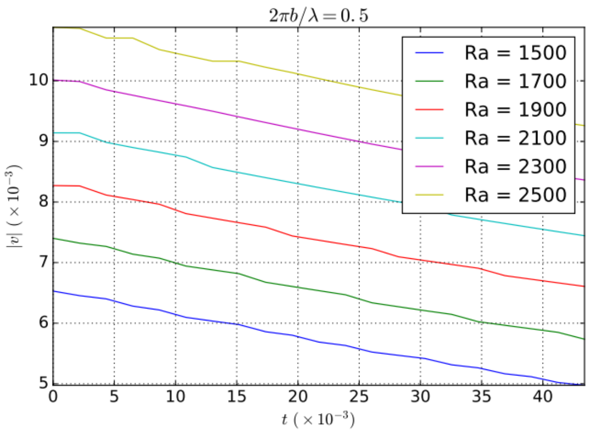

Figure 4 *– Non-dimensional velocity modulus for* $2\pi b/\lambda = 0.5$ *and* $Ra = [1500,1700,1900,2100,2300,2500]$.

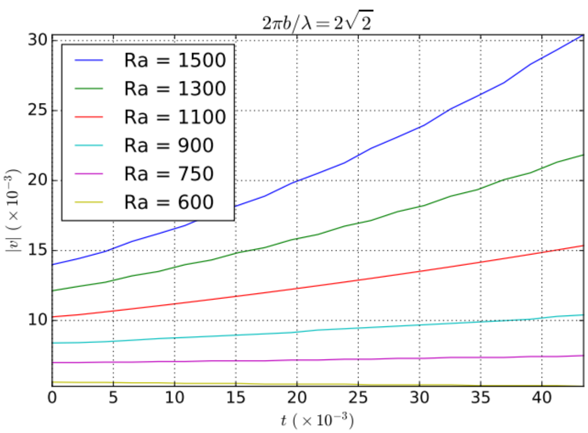

Figure 5 *– Non-dimensional velocity modulus for* $2\pi b/\lambda = 2\sqrt{2}$ *and* $Ra = [600,750,900,1100,1300,1500]$.

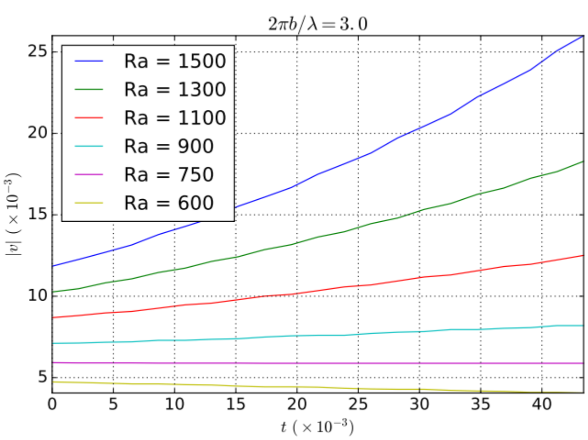

Figure 6 *– Non-dimensional velocity modulus for* $2\pi b/\lambda = 3.0$ *and* $Ra = [600,750,900,1100,1300,1500]$.

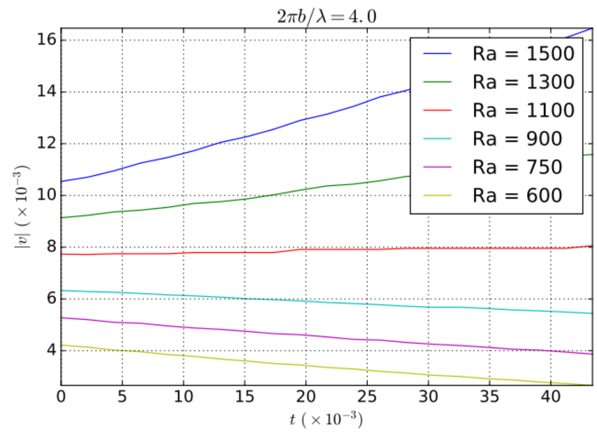

Figure 7 *– Non-dimensional velocity modulus for* $2\pi b/\lambda = 4.0$ *and* $Ra = [600,750,900,1100,1300,1500]$.

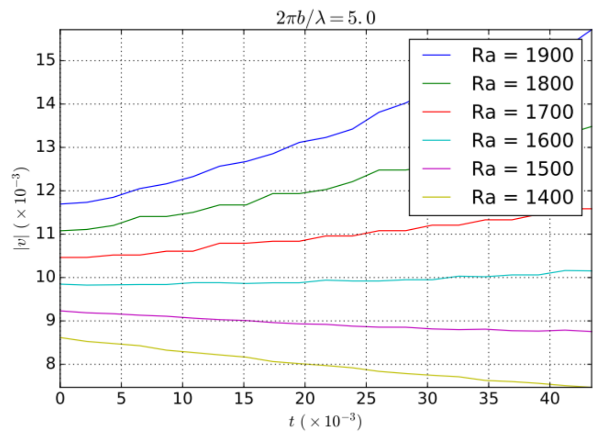

Figure 8 *– Non-dimensional velocity modulus for* $2\pi b/\lambda = 5.0$ *and* $Ra = [1400,1500,1600,1700,1800,1900]$.

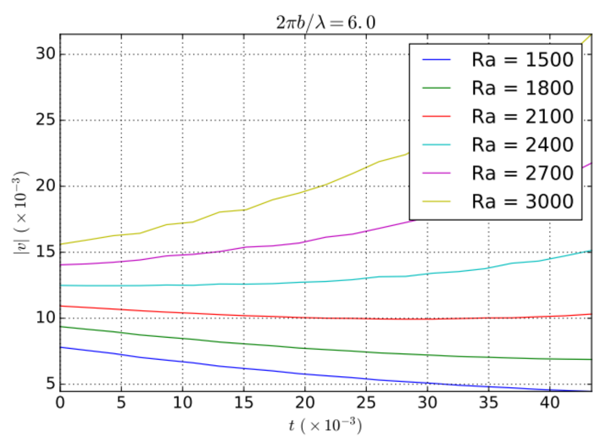

Figure 9 *– Non-dimensional velocity modulus for* $2\pi b/\lambda = 6.0$ *and* $Ra = [1500,1800,2100,2400,2700,3000]$.

All the results of the non-dimensional velocity modulus were analyzed following the criterion: from the $t' = 0$, if the curve decreased, the scenario was considered to be mainly conductive, if the curve increased, the scenarios was considered to be convective. The physical information needed to conclude this it that when we have convection, the stability of the simulation is compromised, increasing the non-dimensional velocity modulus maximum. This information allowed to create the graph in Figure 10, where all the scenarios are summarized by its behavior.

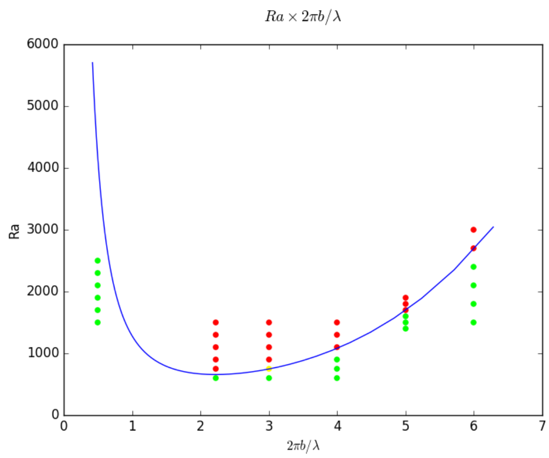

Figure 10 *– Simulated $(2\pi b/\lambda, Ra)$ pairs. The green dots illustrate the predominant conduction scenario found, the red dots illustrate the predominant conductive scenario observed and the yellow dot illustrates the pair with no decrease or increase of the non-dimensional velocity modulus.*

Conclusions

For the studied scenarios, the values that were calculated using the CitcomCU code agreed with the analytical prediction of the heat transfer mechanism for each one of the cases. For the case where no increase or decrease was observed ($2\pi b/\lambda = 3$ and $Ra = 750$), the value was considered too close to the Ra_{CR} to be considered wrong ($Ra_{CR} \approx 747$) and therefore within a reasonable deviation. This allows two conclusions.

The first conclusion is about the agreement between both analytical prediction and numerical approach. This tell us that a discretized medium can represent a continuous medium with good approximation. This kind of conclusion creates reliability for the numerical code which provides the second conclusion.

The second conclusion is about the efficiency of the CitcomCU code as a numerical code to reproduce the suggested scenarios. The observed data of the simulations puts the utilized code as a good code for these types of studies. The only case to be observed with a little more caution is when the Ra of the simulated scenario is very close to the Ra_{CR}. If a very detailed study needs to be made, there is the necessity of increasing the grid size to avoid this deviation. One must say that CitcomCU does not have a constant time variation and increasing the grid size would actually diminish the time between two step and possibly providing enough data to confirm that a decrease would be observed for the $2\pi b/\lambda = 3$ and $Ra = 750$.

References

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